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Hawking Radiation Spectrum and Entropy Correction of Apparent Horizon in a FRW Universe

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Abstract Taking the reaction of the radiation to the spacetime into consideration, we discuss Hawking radiation spectrum and Bekenstein-Hawking entropy correction in Friedmann-Robertson-Walker (FRW) universe by the analytical continuation method. We derive the radiation spectrum that satisfies the unitary principle and the logarithmic correction term of entropy in FRW universe.

Keywords Friedmann-Robertson-Walker universe · Energy conversation · Bekenstein-Hawking entropy correction

1 Introduction

Investigating the black hole Hawking radiation and correction to Bekenstein-Hawking (B-H) entropy is an important subject in theoretical physics. So far there are many methods to calculate Hawking radiation [1–12]. However, all results show that black hole radiation is the black body spectrum. This conclusion has brought a difficult problem to theoretical physicist: is the information conservative in the black hole evaporation process? This black hole information paradox has long been puzzled problem to most theoretical physicists.

In 2000, Parikh and Wilczek proposed tunneling method [13, 14], and in 2005, Robinson and Wilczek developed covariant abnormal method [15]. Recently, by using these two

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methods thermal radiation of black holes is investigated deeply [16–40]. They all derive the outgoing rate of black hole radiation particle is

$$\Gamma = \exp[\Delta S], \tag{1}$$

where ΔS is Bekenstein-Hawking entropy difference before and after the black hole radiation. It satisfies the unitary principle and supports the principle of information conservation. At the same time, there are many methods to discuss the correction to B-H entropy [41, 42]. Most people believe that the correction expression of B-H entropy of Schwarzschild black hole is

$$S = \frac{A}{4G} + \chi \ln \frac{A}{4G}, \tag{2}$$

where A is the area of the black hole horizon, χ is a dimensionless constant. At present, the exact value of logarithmic term coefficient in the correction to black hole B-H entropy is not clear.

Recently, an interesting improvement has already been made by Banerjee and Majhj [43]. They formulated the Hamilton-Jacobi method of tunneling beyond the semiclassical approximation by considering all the terms in the expansion of the one particle action for a scalar particle, and obtained all the higher order quantum correction to the semiclassical results. Some further applications of their method to other black hole, the dynamics of black holes and fermions tunneling also have been done [44, 45]. However, examples given were mostly confined to black hole.

Since we can view a FRW thermodynamical system as a black hole [1], it is of great interest to ask whether there is a Hawking-like temperature associated with the apparent horizon of the FRW universe. Research on thermal property of the FRW universe has aroused great interest [46–54]. Ref. [50] investigated Hawking radiation of apparent horizon in a FRW universe by tunneling method and derived the radiation temperature of the FRW universe [13, 14]. Refs. [52, 53] discussed Fermions tunneling from apparent horizon of the FRW universe. Using the method that Banerjee and Majhj [43] proposed for computing black hole radiation spectrum and correction to radiation temperature. Ref. [54] investigated the thermal radiation of the FRW universe and obtained the revised radiation temperature and B-H entropy correction term correspond to the FRW universe. Ref. [51] discussed the entropy of the FRW universe by brick wall method. At present, the obtained the FRW universe radiation spectrum is a pure thermal spectrum. This does not satisfy the unitary principle. Although Ref. [54] derived the correction term of entropy in the FRW universe, there is a uncertainty dimensional constant factor in the correction term.

In this paper, we extend the method that Ref. [10] investigated black hole radiation and entropy correction and discuss Hawking radiation and B-H entropy correction in the FRW universe. In our calculation, considering the property that radiation wave is a spherical wave, we separate variables in Klein-Gordon equation and induct factor $1/\tilde{r}$ in separation of variables. Under the condition that the total energy is conserved, taking the reaction of the radiation to the spacetime into consideration, we investigate Hawking radiation and B-H entropy correction in the FRW universe by the new defined Tortoise coordinate transformation. We not only derive Hawking radiation spectrum that satisfies the unitary principle but also obtain the logarithmic correction term of B-H entropy that does not contain any constant factor.

2 Friedmann-Robertson-Walker Universe

Let us start with the standard FRW metric

$$ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right). \tag{3}$$

Applying the change of radial coordinate $\tilde{r} = ar$ and the metric (3), one obtains the Painleve-Gulstrand-like metric for the FRW space-time [49]:

$$ds^2 = -\frac{1-\tilde{r}^2/\tilde{r}_A^2}{1-k\tilde{r}^2/a^2}dt^2 - \frac{2H\tilde{r}}{1-k\tilde{r}^2/a^2}dtdr + \frac{1}{1-k\tilde{r}^2/a^2}d\tilde{r}^2 + \tilde{r}^2 d\Omega_2^2, \tag{4}$$

with H is the Hubble parameter, $H = \dot{a}/a$ (the dot represents derivative with respect to the cosmic time t), the radius of the apparent horizon [50, 52, 54]

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}, \tag{5}$$

and Hawking-like temperature

$$T = \frac{1}{2\pi \tilde{r}_A}, \tag{6}$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\varphi^2$ denotes the line element of an unit two-sphere S^2, a is the scale factor of our universe and k is the spatial curvature constant, which can take the values $k = +1$ (positive curvature), $k = 0$ (flat), and $k = -1$ (negative curvature).

3 Hawking Radiation and Entropy Correction

In curved spacetime, Klein-Gordon equation of the particle with rest mass μ_0 is

$$\frac{1}{\sqrt{-g}}\left[\frac{\partial}{\partial x^\mu}\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^\nu}\Phi\right] - \mu_0^2\Phi = 0. \tag{7}$$

From (4), we have

$$\begin{aligned} &g^{tt} = -1, \qquad g^{tr} = -H\tilde{r}, \qquad g^{rr} = 1 - \tilde{r}^2/\tilde{r}_A^2, g^{\theta\theta} = 1/\tilde{r}^2, \\ &g^{\theta\theta} = 1/(\tilde{r}^2\sin^2\theta), \qquad \sqrt{-g} = \frac{\tilde{r}^2\sin\theta}{\sqrt{1-k\tilde{r}^2/a^2}}. \end{aligned} \tag{8}$$

Considering the property that radiation wave is a spherical wave, we separate variables

$$\Phi = \frac{1}{\tilde{r}}\psi(t,\tilde{r})Y_{lm}(\theta,\varphi), \tag{9}$$

whereas $\psi(t,\tilde{r})$ is no longer separable.

$$\frac{\partial^2\psi}{\partial t^2} + \frac{H}{1-k\tilde{r}^2/a^2}\frac{\partial\psi}{\partial t} + 2H\tilde{r}\frac{\partial^2\psi}{\partial t\partial\tilde{r}} + \frac{\tilde{r}(H^2\tilde{r}_A^2 + 1 - k\tilde{r}^2/a^2)}{\tilde{r}_A^2(1-k\tilde{r}^2/a^2)}\frac{\partial\psi}{\partial\tilde{r}} - \left(1 - \frac{\tilde{r}^2}{\tilde{r}_A^2}\right)\frac{\partial^2\psi}{\partial\tilde{r}^2}$$

$$= \left[\mu_0^2 + \lambda - \frac{2}{\tilde{r}^2}\left(1 - \frac{\tilde{r}^2}{\tilde{r}_A^2}\right) - \frac{2}{\tilde{r}^2}\left(\frac{1 - 2\tilde{r}^2/\tilde{r}_A^2 + k\tilde{r}^4/(\tilde{r}_A^2 a^2)}{1 - k\tilde{r}^2/a^2}\right)\right]\psi, \tag{10}$$

λ is the separation constant. After the FRW universe radiated particles with energy ω, E in spacetime line element (4) will be replaced with $E - \omega$. Therefore $\tilde{r}_A$ will be replaced with $\tilde{r}_\omega$ after considering radiation. We define the tortoise coordinate transformation

$$dr_* = \frac{1}{f(\tilde{r})} d\tilde{r}, \tag{11}$$

where $f(\tilde{r}) = 1 - \tilde{r}^2/\tilde{r}_\omega^2$. Substituting (11) into (10), we have

$$\begin{aligned} &\frac{\partial^2\psi}{\partial r_*^2} - 2H\tilde{r}\frac{\partial^2\psi}{\partial t \partial r_*} - \left[\frac{\tilde{r}(H^2\tilde{r}_\omega^2 + 1 - k\tilde{r}^2/a)}{\tilde{r}_\omega^2(1 - k\tilde{r}^2/a^2)} - \frac{2\tilde{r}}{\tilde{r}_\omega^2}\right]\frac{\partial\psi}{\partial r_*} \\ &\quad - f(\tilde{r})\left[\frac{\partial^2\psi}{\partial t^2} + \frac{H}{1 - k\tilde{r}^2/a^2}\frac{\partial\psi}{\partial t}\right] \\ &= f(\tilde{r})\left[\mu_0^2 + \lambda - \frac{2}{\tilde{r}^2}\left(1 - \frac{\tilde{r}^2}{\tilde{r}_\omega^2}\right) - \frac{2}{\tilde{r}^2}\left(\frac{1 - 2\tilde{r}^2/\tilde{r}_\omega^2 + k\tilde{r}^4/(\tilde{r}_\omega^2 a^2)}{1 - k\tilde{r}^2/a^2}\right)\right]\psi. \end{aligned} \tag{12}$$

Near apparent horizon, $f(\tilde{r}_\omega) = 0$. Thus near apparent horizon (12) can be reduced to

$$\frac{\partial^2\psi}{\partial r_*^2} - 2H\tilde{r}\frac{\partial^2\psi}{\partial t \partial r_*} = 0. \tag{13}$$

Now, we assume the frequency to be constant near horizon as like-dependent Vaidya metric in Refs. [51, 55]. So near horizon, when $\tilde{r} \sim \tilde{r}_A \gg 1$, approximate solutions to (13) are [53, 54]

$$\psi_\omega^{out} = \exp[-i\omega t/Hr_\omega\} \exp[-i2\omega r_*], \tag{14}$$

and

$$\psi_\omega^{in} = \exp[-i\omega t/Hr_\omega\}. \tag{15}$$

From (11), near $r = r_\omega$

$$\ln(r_\omega - r) = -\frac{2}{r_\omega} r_* = \kappa_\omega r_*, \tag{16}$$

where $\kappa_\omega = -2/r_\omega$. For

$$r_\omega - r = \exp(\kappa_\omega r_*), \tag{17}$$

thus the solution of outgoing wave of apparent horizon is rewritten as

$$\psi_\omega^{out}(r < r_\omega) = \exp[-i\omega t/Hr_\omega]\,(r_\omega - r)^{-i2\omega/\kappa_\omega}. \tag{18}$$

It is obvious that solution of outgoing wave is singular at apparent horizon surface $r = r_\omega$. Equation (18) only describes the outgoing particles in range $r < r_\omega$ and it can not describe the outgoing particles in range $r > r_\omega$. Taking singularity $r = r_\omega$ as circle center and $|r_\omega - r|$ as radius, we make analytic extension in upper half plane

$$(r_\omega - r) \to |r_\omega - r|e^{-i\pi} = (r - r_\omega)e^{-i\pi}. \tag{19}$$

Thus we derive the outgoing wave in range $r > r_\omega$

$$\psi_\omega^{out}(r > r_\omega) = \exp[-i\omega t/Hr_\omega]e^{-2i\omega r_*}e^{-2\omega\pi/\kappa_\omega}, \tag{20}$$

(20) and (18) describe the outgoing wave of inner and outside apparent horizon respectively. The outgoing rate of outgoing wave on apparent horizon is

$$\Gamma_\omega = \left|\frac{\psi_\omega^{out}(r < r_\omega)}{\psi_\omega^{out}(r > r_\omega)}\right|^2 = e^{4\pi\omega/\kappa_\omega}. \tag{21}$$

The process that the universe radiates particles with energy ω is an integration process [10, 13, 14, 56, 57], that is $\omega = \int_0^\omega d\omega'$, thus the outgoing rate that the universe radiates particles with energy ω is

$$\Gamma(i \to f) = \prod_i \Gamma_{\omega_i} = \exp\left[\int_0^\omega \frac{4\pi d\omega'}{\kappa_{\omega'}}\right] = \exp\left[\int_0^\omega -\frac{d\omega'}{T(r_\omega)}\right], \tag{22}$$

where $T(r_\omega) = \frac{1}{2\pi r_\omega}$. Let the change of the B-H entropy of FRW universe before and after the FRW radiation be as

$$\Delta S = S(E - \omega') - S(E), \tag{23}$$

we obtain

$$\frac{d(\Delta S)}{d\omega'} = \frac{dS(E - \omega')}{d\omega'}, \quad d(\Delta S) = \frac{d(\Delta S)}{d\omega'}d\omega'. \tag{24}$$

Because thermodynamic quantities correspond to FRW universe satisfy the first law of thermodynamics [54]

$$dS = \frac{dE}{T}, \tag{25}$$

we derive

$$d(\Delta S) = -\frac{d\omega'}{T(r_\omega)}. \tag{26}$$

Thus we obtain the outgoing rate that FRW universe radiates particles with energy ω

$$\Gamma(i \to f) = e^{\Delta S}, \tag{27}$$

where ΔS is entropy difference corresponding apparent horizon.

In the radiation rate calculation (21), we do not consider the factor $1/\tilde{r}$ in (9). After we consider this factor $1/\tilde{r}$, (27) of the outgoing rate that FRW universe apparent horizon radiates particles with energy ω should be written as

$$\Gamma(i \to f) = \frac{r_i^2}{r_f^2}e^{\Delta S} = \exp\left[\left(\frac{A_f}{4} - \ln\frac{A_f}{4}\right) - \left(\frac{A_i}{4} - \ln\frac{A_i}{4}\right)\right]. \tag{28}$$

Thus the first order correction to B-H entropy corresponding FRW universe apparent horizon is

$$S = \frac{A}{4} - \ln\frac{A}{4}, \tag{29}$$

where A is the FRW universe apparent horizon area.

Compared (29) with (2), we derive $\chi = -1$ in (2). So we obtain the correction to B-H entropy in FRW universe.

According to quantum mechanics, the probability of transition from initial state to the final state can be expressed as

$$\Gamma(i \to f) = \left|M_{fi}\right|^2 A, \tag{30}$$

where A is phase factor, $|M_{fi}|^2$ is the square of the probability in transition process. Since the number of final state can be expressed as e^- exponential form of the final state entropy, similarly the number of initial state can be expressed as e^- exponential form of the initial state entropy. Thus we have

$$\Gamma(i - f) = \frac{e^{S_f}}{e^{S_i}} = e^{\Delta S}, \tag{31}$$

which is consistent with (28). Compared with the standard form of quantum mechanics, the difference is only one factor.

4 Conclusion

We extend the Damour-Ruffini method that has been used to discuss Hawking radiation and investigate radiation spectrum of FRW universe under the condition that the energy is conservative and the reaction of the radiation of particles to the spacetime exists. The radiation spectrum satisfies the unitary principle.

In our calculation, considering the property that radiation wave is a spherical wave, we introduce factor $1/\tilde{r}$ in (9). For introduction of this factor, we naturally derive B-H entropy correction term in FRW universe. Ref. [54] investigated correction to B-H entropy in FRW universe using the method proposed by Banerjee and Majhj [43]. They derived the logarithmic correction term of entropy. However, in the given result there are an uncertainty factor and dimension constant factor α_1. Through our research, we derive the specific value of dimension constant factor α_1, $\alpha_1 = -1/\pi$.

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